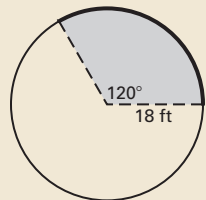


Extra Example 4

Children at a day camp are playing a game on a circular field. The shaded sector in the figure is called the “safe zone,” and is marked off by rope along its outer edge. Find the length of the rope and the area of the safe zone.



about 37.7 ft; about 339 ft²

Key Question to Ask for Example 4

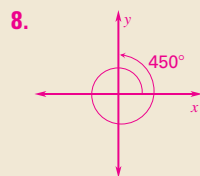
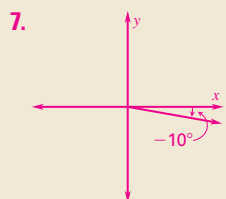
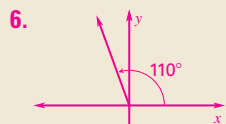
- How does arc length relate to the circumference of the entire circle? **The circumference of the complete circle is 4 times the arc length.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: What is radian measure?

- Angles with positive measure result from counterclockwise rotation; negative measures result from clockwise rotation.**
- The formulas for arc length and area of a sector of a circle use radian measure for the central angle.**

Radian measure and degree measure are two different systems for measuring angles. One radian is the measure of an angle in standard position whose terminal sides intercept an arc whose length is equal to the radius of the circle.



EXAMPLE 4 Solve a multi-step problem

SOFTBALL A softball field forms a sector with the dimensions shown. Find the length of the outfield fence and the area of the field.

Solution

STEP 1 Convert the measure of the central angle to radians.

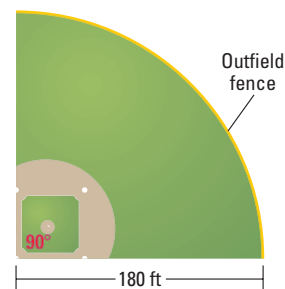
$$90^\circ = 90^\circ \left(\frac{\pi \text{ radians}}{180^\circ} \right) = \frac{\pi}{2} \text{ radians}$$

STEP 2 Find the arc length and the area of the sector.

$$\text{Arc length: } s = r\theta = 180 \left(\frac{\pi}{2} \right) = 90\pi \approx 283 \text{ feet}$$

$$\text{Area: } A = \frac{1}{2}r^2\theta = \frac{1}{2}(180)^2 \left(\frac{\pi}{2} \right) = 8100\pi \approx 25,400 \text{ ft}^2$$

► The length of the outfield fence is about 283 feet. The area of the field is about 25,400 square feet.



AVOID ERRORS

You must write the measure of an angle in radians when using the formulas for the arc length and area of a sector.



GUIDED PRACTICE for Example 4

9. **WHAT IF?** In Example 4, estimate the length of the outfield fence and the area of the field if the outfield fence is 220 feet from home plate.

about 346 ft, about 38,013 ft²

13.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 11, 23, and 51

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 14, 31, 50, and 53

SKILL PRACTICE

A

- VOCABULARY** Copy and complete: An angle is in standard position if its vertex is at the ? and its ? lies on the positive x-axis. **origin, initial side**
- ★ WRITING** How does the sign of an angle's measure determine its direction of rotation? **If the angle is positive you travel counter-clockwise from the initial position on the x-axis. If the angle is negative you travel clockwise from the initial position.**

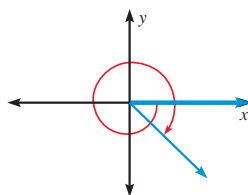
VISUAL THINKING Match the angle measure with the angle.

3. -240° **B**

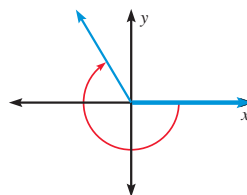
4. 600° **C**

5. $-\frac{9\pi}{4}$ **A**

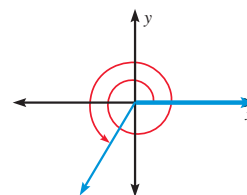
A.



B.



C.



Differentiated Instruction

Visual Learners Suggest that students create a visual representation of the softball field described in **Example 4**. Have students superimpose a coordinate plane over their drawing, and then have them label the initial side, vertex, terminal side, and label the angle measure. Such a visual representation will help students set up their calculations correctly and calculate the proper solution.

See also the *Algebra 2 Toolkit* for more strategies.

DRAWING ANGLES Draw an angle with the given measure in standard position.

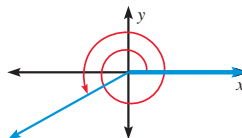
6–13. See margin.

6. 110° 7. -10° 8. 450° 9. -900°

10. 6π 11. $\frac{5\pi}{18}$ 12. $-\frac{5\pi}{3}$ 13. $\frac{26\pi}{9}$

14. ★ **MULTIPLE CHOICE** Which angle measure is shown in the diagram? **C**

- (A) -150° (B) 210°
(C) 570° (D) 930°



EXAMPLES
2 and 3

on pp. 860–861
for Exs. 15–22

EXAMPLE 3

on p. 861
for Exs. 23–31

EXAMPLE 4 B

on p. 862
for Exs. 32–38

HINT

For Exs. 39–46,
set your
calculator in
radian mode.

FINDING COTERMINAL ANGLES Find one positive angle and one negative angle that are coterminal with the given angle. 15–22. Sample answers are given.

15. 70° **$430^\circ, -290^\circ$** 16. 255° **$615^\circ, -105^\circ$** 17. -125° **$235^\circ, -485^\circ$** 18. 820° **$100^\circ, -260^\circ$**

19. $\frac{9\pi}{2}$ **$\frac{\pi}{2}, -\frac{3\pi}{2}$** 20. $-\frac{7\pi}{6}$ **$\frac{5\pi}{6}, -\frac{19\pi}{6}$** 21. $\frac{28\pi}{9}$ **$\frac{10\pi}{9}, -\frac{8\pi}{9}$** 22. $\frac{20\pi}{3}$ **$\frac{2\pi}{3}, -\frac{4\pi}{3}$**

CONVERTING MEASURES Convert the degree measure to radians or the radian measure to degrees.

23. 40° **$\frac{2\pi}{9}$** 24. 315° **$\frac{7\pi}{4}$** 25. -260° **$-\frac{13\pi}{9}$** 26. 500° **$\frac{25\pi}{9}$**

27. $\frac{\pi}{9}$ **20°** 28. $-\frac{\pi}{4}$ **-45°** 29. 5π **900°** 30. $\frac{14\pi}{15}$ **168°**

31. ★ **MULTIPLE CHOICE** Which angle measure is equivalent to $\frac{13\pi}{6}$ radians? **B**

- (A) 30° (B) 390° (C) 750° (D) 1110°

FINDING ARC LENGTH AND AREA Find the arc length and area of a sector with the given radius r and central angle θ .

32. $r = 4$ in., $\theta = \frac{\pi}{6}$ 33. $r = 3$ m, $\theta = \frac{5\pi}{12}$ 34. $r = 15$ cm, $\theta = 45^\circ$
about 2.09 in., about 4.19 in.² about 3.93 m, about 5.89 m² about 11.8 cm, about 88.4 cm²
35. $r = 12$ ft, $\theta = 150^\circ$ 36. $r = 18$ m, $\theta = 25^\circ$ 37. $r = 25$ in., $\theta = 270^\circ$
about 31.4 ft, about 188 ft² about 7.85 m, about 70.6 m² about 118 in., about 1470 in.²

38. **ERROR ANALYSIS** Describe and correct the error in finding the area of a sector with a radius of 6 centimeters and a central angle of 40° .

$$A = \frac{1}{2}(6)^2(40) = 720 \text{ cm}^2$$

The angle measure must be in radians before using the formula; $A = \frac{1}{2}(6)^2\left(\frac{2\pi}{9}\right) \approx 12.6 \text{ cm}^2$.

EVALUATING FUNCTIONS Evaluate the trigonometric function using a calculator if necessary. If possible, give an exact answer.

39. $\cos \frac{\pi}{3}$ **$\frac{1}{2}$** 40. $\sin \frac{\pi}{4}$ **$\frac{\sqrt{2}}{2}$** 41. $\tan \frac{\pi}{6}$ **$\frac{\sqrt{3}}{3}$** 42. $\sec \frac{\pi}{9}$ **about 1.06**
43. $\cot \frac{\pi}{8}$ **about 2.41** 44. $\cos \frac{\pi}{6}$ **$\frac{\sqrt{3}}{2}$** 45. $\sin \frac{3\pi}{7}$ **about 0.975** 46. $\csc \frac{4\pi}{15}$ **about 1.35**

- C** 47. **CHALLENGE** A rotating object that passes through an angle θ during time t has an angular velocity v given by the formula $v = \frac{\theta}{t}$. Find the angular velocity of the hour hand, the minute hand, and the second hand on a 12 hour clock. Give all answers in degrees per hour. **30° per h, 360° per h, $21,600^\circ$ per h**

4 PRACTICE AND APPLY

Assignment Guide

Answer Transparencies
available for all exercises

Basic:

Day 1: pp. 862–865
Exs. 1–11, 14–18, 23–37 odd, 38,
48–51, 55, 57, 59, 63, 66, 70

Average:

Day 1: pp. 862–865
Exs. 1–5, 9–11, 14, 18–20, 25–28,
31–45 odd, 48–53, 56, 58, 60, 64,
67, 72

Advanced:

Day 1: pp. 862–865
Exs. 1, 2, 11–14, 20–22, 28–31,
35–54*, 62, 65, 69, 73

Block:

pp. 862–865
Exs. 1–5, 9–11, 14, 18–20, 25–28,
31–45 odd, 48–53, 56, 58, 60, 64, 67,
72 (with 13.1)

Differentiated Instruction

See *Algebra 2 Best Practices Toolkit*
for suggestions on addressing the
needs of a diverse classroom.

Homework Check

For a quick check of student under-
standing of key concepts, go over
the following exercises:

Basic: 8, 16, 24, 32, 48

Average: 10, 18, 26, 34, 50

Advanced: 12, 22, 28, 36, 52

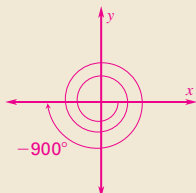
Extra Practice

- Student Edition, p. 1022
- Chapter 13 Resource Book:
Practice levels A, B, C, pp. 16–18

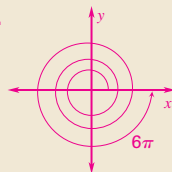
Practice Worksheet

An easily-readable reduced
practice page (with answers)
for this lesson can be found
on p. 850C.

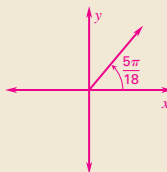
9.



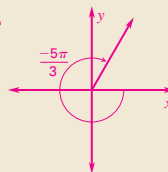
10.



11.



12.



13.

